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transport through a constriction in a FQH annulus

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The flux periodicity of thermodynamic properties of an annulus in the fractional quantum hall state with a constriction is considered. It is found that ϕ_0 - periodicity is obtained due to transfer of fractionally charged particles or composite fermions between the edges of the annulus, respectively. The result for the finite magnitude of the persistent current across a very strong constriction is presented, as obtained with an extension of Wen's edge state theory.

Keywords: Fractional quantum Hall effect, Persistent Current, Tunneling

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I. INTRODUCTION

The Fractional Qunatum Hall effect, discovered by Stoermer, Tsui and Gossard, [1] was explained by Laughlin in a theory based on a trial wave function [2]. Since then, the question was debated if quasiparticle excitations with fractional charge and statistics can be observed from such a state.

Thouless and Gefen considered an annulus in a strong magnetic field which condenses the electrons to a fractional quantum hall state. They predicted that thermodynamic properties of such a system have flux periodicity ϕ_0 due to the existence of families of states of the FQH- annulus. These were connected by a physical mechanism, the finite back scattering amplitude of fractionally charged quasiparticles [3].

Recently, this was verified by deriving the persistent current in a FQH- annulus with a weak constriction [4]. There, an extension of Wen's edge state theory [5] was used, which takes into account the dynamics of zero- modes [6].

There exists the alternative theory of the fractional quantum hall effect based on composite fermions as proposed by Jain [7].

A composite Fermion of a FQH state of filling factor $\nu = 1/(2m+1)$ has charge (-e), and 2m vortices attached to it, corresponding to 2 m flux quanta $\phi_0 = 2\pi c/e$ aligned oppositely to the external magnetic field B.

II. THE FQH ANNULUS IN THE COMPOSITE FERMION PICTURE

Let us consider the annulus of a FQH liquid within the theory of composite fermions. When a composite fermion is moved around a circle in the annulus, which is pierced by an additional flux ϕ, it aquires a phase

$$\chi = \frac{2\pi}{\phi_0} \oint drA = \frac{SB}{\phi_0} + \frac{\phi}{\phi_0} - 4m\pi x, \quad (1)$$

where S is the area enclosed by the circle [8]. x is the number of composite fermions within that circle. Since there are no composite fermions in the inner area S_i of the annulus, we have $x = \nu(S - S_i)B/\phi_0$, giving

$$\chi = 2\pi \frac{S\tilde{B}}{\phi_0} + 2\pi \frac{\bar{\phi}}{\phi_0}, \quad (2)$$

so that the composite fermion sees a reduced magnetic field

$$\tilde{B} = (1 - 2m\nu)B \quad (3)$$

and an enhanced flux, piercing the annulus,

$$\bar{\phi} = \phi + 2m\nu S_i B. \quad (4)$$

We obtain the guiding centers of the Eigenstates of the CF's by demanding quantization of the phase $\chi = 2\pi l$, l an integer [9].

An adiabatic increase of the flux ϕ reduces the positions of the CF's. Thus, the FQH liquid moves up the inner edge. The area S_i where there are no CF's becomes smaller, so that there are δx_i CF's added to the inner edge with,

$$\delta x_i = \nu \frac{\delta\phi}{\phi_0}. \quad (5)$$

An equal amount of CF's is removed from the outer edge. After a change of the flux by one flux quantum ϕ_0, a fraction ν of a CF is added to the inner edge [10]. This is in agreement with the fact that the Laughlin wave function yields the addition of a charge $-\nu e$ to the inner edge, when one flux quantum is added adiabatically into the annulus. If only electrons could tunnel between the edges of the FQH annulus, its periodicity would be enhanced to $1/\nu\phi_0$.

Is the ϕ_0- periodicity as required by the Byers- and Young theorem [11] restored by the transport of composite fermions between the edges?

The removal of a CF from the inner edge adds 2m flux quanta to the flux piercing the annulus, since the two vortices attached to the removed electron did cancel 2 flux quanta of the magnetic field B. Thus, the total number of composite fermions added to the inner edge, when the flux piercing the annulus is changed by $\delta\phi$, and one composite fermion tunnels to the outer edge, is:

$$\delta x_i = \nu \frac{\delta\phi + 2m\phi_0}{\phi_0} - 1. \quad (6)$$

Thus, after a flux change of 1 flux quantum, the tunneling of the composite fermion to the outer edge brought the system to its state at $\phi = 0$, restoring the ϕ_0- periodicity.

Therefore, in order that the flux periodicity of the FQH- annulus can be understood from the composite fermion perspective, there has to be tunneling of composite fermions between the edges of the annulus.

It is now natural to ask if the tunneling amplitude of composite fermions is renormalized, how this is compared with the renormalization of fractionally charged particles [12], and if it yields the same temperature and size dependence of the persistent current in the presence of a constriction as was obtained in Ref. [4], using the edge state theory

of fractionally charged particles. To this end, one has to formulate the edge state theory of composite fermions which is complicated by the dynamics of the vortices attached to the fermions. Then, one can use this theory to find a bosonized form of composite edge fermions. This is the subject of ongoing research.

Here, rather we want to conclude by presenting results on the persistent current through a very strong constriction using the edge state theory as presented in Ref. [4].

III. PERSISTENT CURRENT THROUGH A VERY STRONG CONSTRICTION

We consider again an annulus in the FQH state at odd inverse filling factor. There is a strong constriction at one point in the annulus, producing a weak link through which particles can tunnel, only. The potential barrier is taken to be much larger than the energy gap in the bulk of the fractional quantum hall liquid, $V_0 >> \Delta_{bulk}$. Then, the many- body wave function of the FQH state in the annulus decays exponentially. Thus, the strong correlations in the FQH state can not extend through it and only electrons can tunnel across the barrier.

We obtain the persistent current in second order perturbation theory in the most relevant electron tunneling amplitude $w_{1/\nu}$. We use the edge state theory of a single chiral edge connected at the constriction.

$$I(\varphi) = -\frac{e}{\pi} \sum_{ang.mom.l} \partial_\varphi w_{1/\nu eff}(l,\varphi) \cos[2\pi(l+\varphi)], \tag{7}$$

where $\varphi = \phi/\phi_0$, ϕ being the flux penetrating the annulus in addition to the constant background magnetic field B. . For temperatures below the level spacing, $T \ll v/L$, the renormalized tunneling amplitude of electrons w_{eff} is given by

$$w_{1/\nu}(l,\varphi)_{eff} = w_{1/\nu}(l,\varphi)(\pi v/(2L\Lambda))^{1/\nu} (2(1-\cos(\pi(x_R-x_L)/L)))^{-1/(2\nu)}. \tag{8}$$

Here, L is the total length of the single chiral edge, and x_R and x_L are the two positions at which the tunneling takes place. Their distance is taken to be of the order of $L/2$. $w(l,\varphi)$ are the flux dependent coefficients of an expansion of the tunneling amplitude of electrons across the barrier in terms

of the Eigen states of angular momentum l of the annulus of noninteracting electrons without constriction. They satisfy the relation, $w_{1/\nu}(l,\varphi+1) = w_{1/\nu}(l+1,\varphi)$.

Thus, there is in the presence of a very strong constriction a finite ϕ_0- periodic persistent current of electrons across the weak link and its amplitude decreases with increasing circumference L of the annulus only slowly, like a power law.

When the temperature exceeds the level spacing, $T \gg v/L$, the effective tunneling amplitude is

$$
\begin{aligned}
&w_{1/\nu eff}(l,\varphi) = \\
&w_{1/\nu}(l,\varphi)(\pi T/\Lambda)^{1/\nu} \exp[-1/\nu TL/v \\
&(1-\exp(-\pi \mid x_R - x_L \mid /L)) \cos(\pi v/(2TL))] \\
&(1-\exp(-2\pi T/v \mid x_R - x_L \mid))^{-1/\nu} \qquad (9)
\end{aligned}
$$

and the persistent current has an exponentially small amplitude.

The derivation, as well as the result for a potential barrier smaller than the bulk energy gap, $V_0 < \Delta_{bulk}$ will be presented elsewhere.

IV. CONCLUSION

The flux periodicity of a FQH annulus can be understood within the picture of composite fermions. The flux periodicty ϕ_0 is restored due to the existence of back scattering of composite fermions between the edges.

There is a finite persistent current of electrons across a very strong constriction, when the barrier height exceeds the bulk energy gap, at temperatures not exceeding the level spacing.

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